

2.1 Exercises

State the quadrant in which each point lies.

- (1, -2), (-2, 1), (3, 4), (-5, -6)
- $(\pi, 2)$, $(3, -\sqrt{2})$, $(4, 0)$, $(-\sqrt{3}, \sqrt{3})$

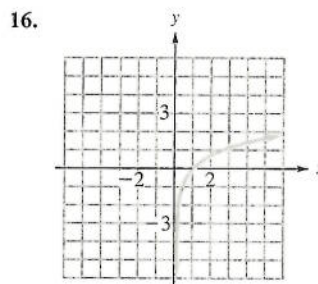
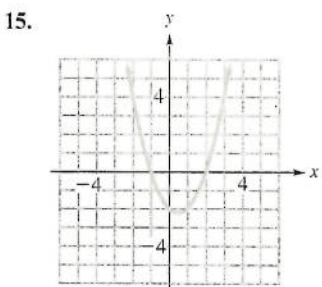
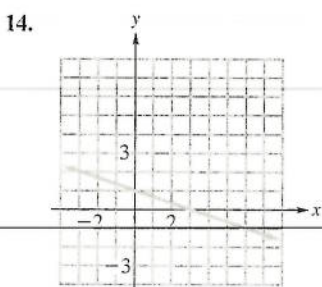
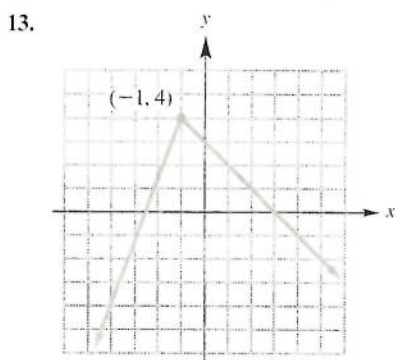
Determine whether the given ordered pair is a solution of the given equation. (See Example 2.)

- (1, -3); $3x - y - 6 = 0$
- (2, -1); $x^2 + y^2 - 6x + 8y = -15$
- (3, 4); $(x - 2)^2 + (y + 2)^2 = 6$
- (1, -1); $\frac{x^2}{2} + \frac{y^2}{3} = -4$

Sketch the graph of each of these equations. (See Example 3.)

- $4y + 3x = 12$
- $2x + 7y = 14$
- $8x + 3y = 12$
- $9y - 4x = 12$
- $x = 2y + 3$
- $x - 3y = 0$

List the x -intercepts and y -intercepts of each graph.



Find the x -intercepts and y -intercepts of the graph of each equation. You need not sketch the graph. (See Example 4.)

- $3x + 4y = 12$
- $x - 2y = 5$
- $2x - 3y = 24$
- $3x + y = 4$
- $y = x^2 - 9$
- $y = x^2 + 4$
- $y = x^2 + x - 20$
- $y = 5x^2 + 6x + 1$
- $y = 2x^2 - 5x + 7$
- $y = 3x^2 + 4x - 4$

Sketch the graph of the equation. (See Examples 3–5.)

- $y = x^2$
- $y = x^2 + 2$
- $y = x^2 - 3$
- $y = 2x^2$
- $y = x^2 - 6x + 5$
- $y = x^2 + 2x - 3$
- $y = x^3$
- $y = x^3 - 3$
- $y = \sqrt{x + 4}$
- $y = \sqrt{x - 2}$

Sketch the graph of the information in the following tables. (See Example 1.)

37. **Military Construction** The following table gives the amount of annual spending in the U.S. defense budget (in billions of dollars) for military construction. Let $x = 12$ correspond to the year 2012. (Data from: U.S. Office of Management and Budget.)

Year	2012	2013	2014	2015	2016
Spending	11.4	8.1	8.4	5.7	7.0

38. **Military Procurement** The following table gives the amount of annual spending in the U.S. defense budget (in billions of dollars) for military procurement. Let $x = 12$ correspond to the year 2012. (Data from: U.S. Office of Management and Budget.)

Year	2012	2013	2014	2015	2016
Spending	118	98	100	103	115

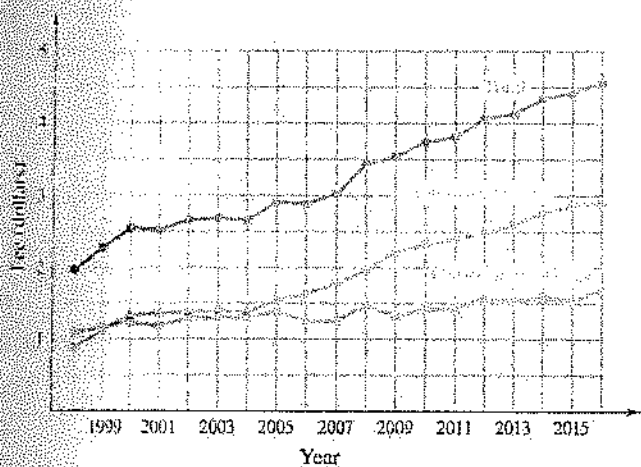
39. **Coal Mine Fatalities** The number of fatalities in coal mines in the United States for recent years is given in the following table. Let $x = 10$ correspond to the year 2010. (Data from: U.S. Mine Safety and Health Administration.)

Year	2010	2011	2012	2013	2014	2015	2016
Number	48	20	20	20	16	12	9

40. **All Mining Fatalities** The number of fatalities in all mining activities in the United States for recent years is given in the table. Let $x = 10$ correspond to the year 2010. (Data from: U.S. Mine Safety and Health Administration.)

Year	2010	2011	2012	2013	2014	2015	2016
Number	71	36	36	42	46	29	26

ATM Fees An article in *The Wall Street Journal* on October 4, 2015, spoke of rising automated teller (ATM) fees. The graph below shows the average costs of noncustomer, out-of-network, and total fees from 1998 to 2016. (Data from: *The Wall Street Journal* and *Bankrate.com*.) (See Examples 6 and 7.)



41. In what year were noncustomer and out-of-network fees almost the same amount?
42. In what year did the average out-of-network fee exceed \$1.50 for the first time?
43. Which year was the last year that the average noncustomer fee was below \$2.50?
44. Which year marked the first time that the average total fees rose above \$3.00?

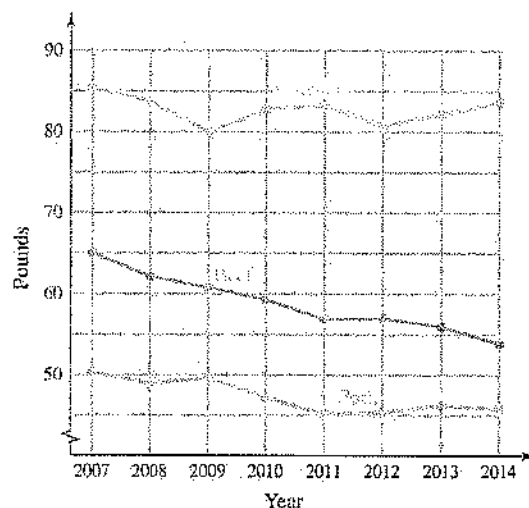
Business Use the revenue and cost graphs for the Webster Cell Phone Company in Example 7 to do Exercises 45–48.

45. Find the approximate cost of manufacturing the given number of phones.
- (a) 20,000 (b) 36,000 (c) 48,000
46. Find the approximate revenue from selling the given number of phones.
- (a) 12,000 (b) 24,000 (c) 36,000
47. Find the approximate profit from manufacturing the given number of phones.
- (a) 20,000 (b) 28,000 (c) 36,000
48. The company must replace its aging machinery with better, but much more expensive, machines. In addition, raw material prices increase, so that monthly costs go up by \$250,000.

Owing to competitive pressure, phone prices cannot be increased, so revenue remains the same. Under these new circumstances, find the approximate profit from manufacturing the given number of phones.

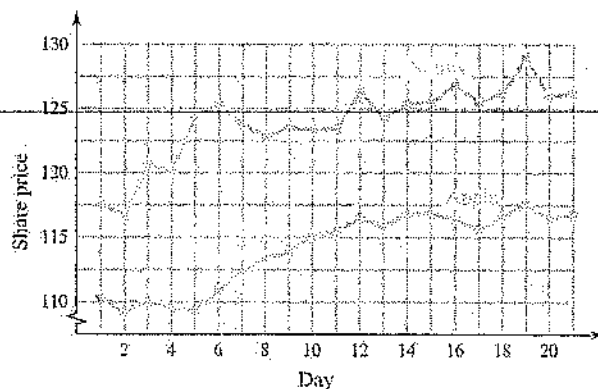
- (a) 20,000 (b) 36,000 (c) 40,000

Business The graph below gives the annual per-person retail availability of beef, chicken, and pork (in pounds) from 2007 to 2014. Use the graph to answer the following questions. (Data from: U.S. Department of Agriculture.)



49. What is the approximate annual per-person retail availability of beef, chicken, and pork in 2012?
50. In what year was annual availability of beef the highest?
51. In what year was annual availability of chicken the lowest?
52. How much did the annual availability of beef decrease from 2007 to 2014?

Netflix and Apple Stock Prices The graph below shows the opening share prices (in dollars) for Netflix, Inc., and Apple Inc. for the 21 trading days in December 2016. Use this graph to answer Exercises 53–62.



53. What is the approximate share price of Apple stock on day 10?
54. What is the approximate share price of Netflix stock on day 8?

55. On which days is the share price of Netflix stock above \$125?
56. On which days is the share price of Apple stock below \$115?
57. On what day was the Apple share price at its highest, and what was the value?
58. On what day was the Netflix share price at its highest, and what was the value?
59. On what day was the Netflix share price at its lowest, and what was the value?
60. On what day was the Apple share price at its lowest, and what was the value?
61. Over the course of the month, what was the gain in share price for Netflix?
62. Over the course of the month, what was the gain in share price for Apple?

Use a graphing calculator to find the graph of the equation. (See Example 8.)

63. $y = x^2 + x + 1$
64. $y = 2 - x - x^2$
65. $y = (x - 3)^3$
66. $y = x^3 + 2x^2 + 2$
67. $y = x^3 - 3x^2 + x - 1$
68. $y = x^4 - 5x^2 + 2$

Use a graphing calculator for Exercises 69–70.

69. Graph $y = x^4 - 2x^3 + 2x$ in a window with $-3 \leq x \leq 3$. Is the “flat” part of the graph near $x = 1$ really a horizontal line segment? (Hint: Use the trace feature to move along the “flat” part and watch the y -coordinates. Do they remain the same [as they should on a horizontal segment]?)
70. (a) Graph $y = x^4 - 2x^3 + 2x$ in the standard window (the one with $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$). Use the trace feature to approximate the coordinates of the lowest point on the graph.
- (b) Use a minimum finder to obtain an accurate approximation of the lowest point. How does this compare with your answer in part (a)?

Use a graphing calculator to approximate all real solutions of the equation. (See Example 9.)

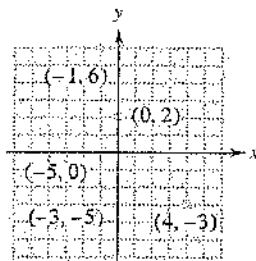
71. $x^3 - 3x^2 + 5 = 0$
72. $x^3 + x - 1 = 0$
73. $2x^3 - 4x^2 + x - 3 = 0$
74. $6x^3 - 5x^2 + 3x - 2 = 0$

75. $x^5 - 6x + 6 = 0$

76. $x^3 - 3x^2 + x - 1 = 0$

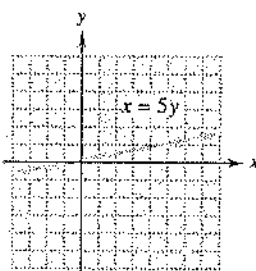
Checkpoint Answers

1.



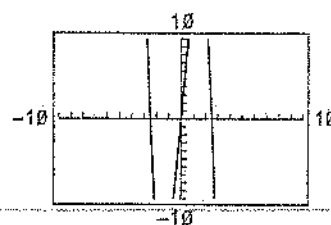
2. (a) and (c)

3.

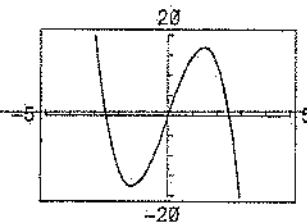


4. (a) x -intercept 4, y -intercept 3
(b) x -intercept $8/5$, y -intercept -4
5. The highest was 2272 on Day 9
The lowest was 2191 on Days 1 and 2
6. (a) About \$1,200,000 (rounded)
(b) About $-\$500,000$ (that is, a loss of \$500,000)

7. (a)



(b)



8. $x \approx -2.330059$

2.2 Equations of Lines

Straight lines, which are the simplest graphs, play an important role in a wide variety of applications. They are considered here from both a geometric and an algebraic point of view.

Find an equation of the line satisfying the given conditions.

53. Through the origin with slope 5
54. Through the origin and horizontal
55. Through (6, 8) and vertical
56. Through (7, 9) and parallel to $y = 6$
57. Through (3, 4) and parallel to $4x - 2y = 5$
58. Through (6, 8) and perpendicular to $y = 2x - 3$
59. x -intercept 6; y -intercept -6
60. Through $(-5, 2)$ and parallel to the line through (1, 2) and (4, 3)
61. Through $(-1, 3)$ and perpendicular to the line through (0, 1) and (2, 3)
62. y -intercept 3 and perpendicular to $2x - y + 6 = 0$

Depreciation The lost value of equipment over a period of time is called depreciation. The simplest method for calculating depreciation is straight-line depreciation. The annual straight-line depreciation D of an item that cost x dollars with a useful life of n years is $D = (1/n)x$. Find the depreciation for items with the given characteristics.

63. Cost: \$15,965; life 12 yr
64. Cost: \$41,762; life 15 yr
65. Cost: \$201,457; life 30 yr
66. Rai Corp. has an incentive compensation plan under which a branch manager receives 10% of the branch's income after deduction of the bonus, but before deduction of income tax. The income of a particular branch before the bonus and income tax was \$165,000. The tax rate was 30%. What was the amount of the bonus?

Light Beer Sales Data from *The Wall Street Journal* indicate that the percent change in beer shipments since the year 2007 for Miller Lite beer can be approximated by

$$y = -3.9x + 27.3,$$

where $x = 7$ corresponds to the year 2007. Find the percent change since 2007 that occurred by the following years.

67. 2010
68. 2014

Assume the model remains accurate, and find the first full year in which the percent change reaches the following values.

69. -32%
70. -40%

Global Malaria Cases Data from *The Wall Street Journal* indicate the number of global malaria cases has risen sharply since the year 2000. The equation

$$y = 5.6x + 52$$

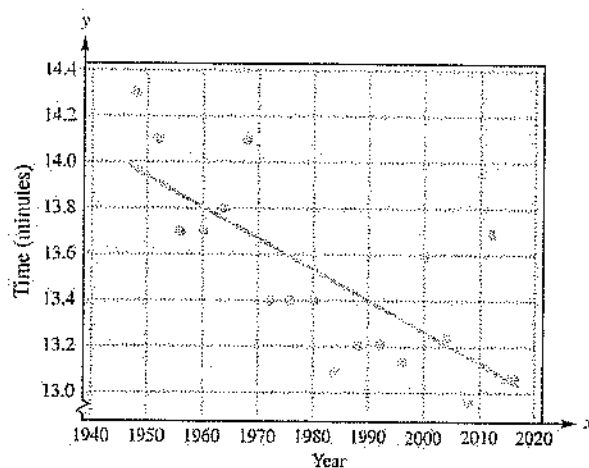
approximates the number of global malaria cases y (in millions), where $x = 0$ corresponds to the year 2000. Find the number of global malaria cases in the following years.

71. 2007
72. 2015

Assume the model remains accurate, and find the first full year in which the number of global malaria cases per year achieves the following values.

73. 150 million
74. 175 million

Olympic 5000-Meters The accompanying graph shows the winning time (in minutes) at the Olympic Games from 1952 to 2016 for the men's 5000-meter run, together with a linear approximation of these data. (Data from: *The World Almanac and Book of Facts; 2017.*)



The equation for the linear approximation is $y = -.0135x + 40.32$, where x is the actual year.

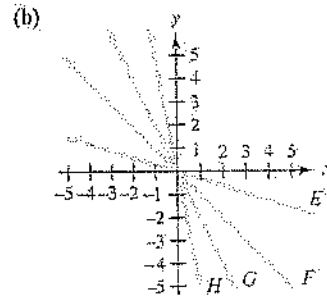
75. What does the linear approximation predict for the winning time for the 2016 Olympics for this event? The actual time was 13.06 minutes. How close was the mathematical model's estimate?
76. What does the slope of the line represent? Why is it negative?
77. **Luxury Handbags** Michael Kors now dominates the handbag market in North America. In 2011, the company had 5% of the market; in 2015, it had 23%. (Data from: *The Wall Street Journal.*)
 - (a) Write a linear equation expressing the percentage of the luxury handbag market y that Michael Kors had x years after 2000.
 - (b) What was the percentage of the market that Michael Kors had in 2013?
 - (c) If the trend continues, what will be the first full year in which Michael Kors had 30% of the market?
78. **Cocoa Production** In 2005, global cocoa bean production was approximately 3.3 million metric tons. In 2015, production was approximately 4.1 million metric tons. (Data from: *The Wall Street Journal.*)
 - (a) Write a linear equation expressing cocoa bean production y in terms of the number of years x after 2000.
 - (b) What was the cocoa bean production in 2012?
 - (c) If the trend continues, what will be the first full year in which cocoa bean production exceeds 5 million metric tons?

79. **Employee Health Costs** In the year 2010, the average annual cost to employees for employer family health coverage was approximately \$3500. In 2015, that amount was approximately \$5000. (Data from: *The Wall Street Journal*.)

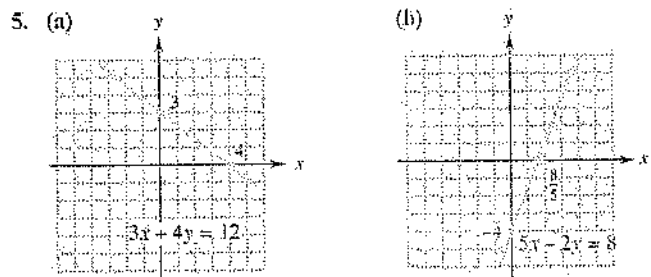
- Write a linear equation expressing the average annual cost to employees y in terms of the number of years x after 2000.
- What was the average annual cost in 2014?
- If the trend continues, what will be the first full year in which costs exceed \$6000?

80. **Employee Health Costs** In the year 2010, the average annual cost to employers for employer family health coverage was approximately \$10,000. In 2015, that amount was approximately \$12,000. (Data from: *The Wall Street Journal*.)

- Write a linear equation expressing the average annual cost to employers y in terms of the number of years x after 2000.
- What was the average annual cost in 2012?
- If the trend continues, what will be the first full year in which costs exceed \$25,000?



- (c) The larger the slope in absolute value, the more steeply the line falls from left to right.



Checkpoint Answers

1. (a) 6.5 (b) $-9/2$

2. (a) $y = \frac{2}{3}x - 3$ (b) $y = -\frac{3}{2}x + \frac{1}{4}$

3. (a) Slope $-1/4$; y -intercept $3/2$

(b) Slope 3; 2; y -intercept $-1/2$

4. (a) Slope of $E = -3$; slope of $F = -1$; slope of $G = -2$;
slope of $H = -5$.

6. (a) Perpendicular (b) Neither (c) Parallel

7. (a) $y + 2 = -\frac{3}{5}(x - 5)$; $y = -\frac{3}{5}x + 1$

(b) $y - 8 = \frac{1}{3}(x - 6)$; $y = \frac{1}{3}x + 6$

8. (a) $y = -\frac{1}{2}x + 4$

(b) $y = -\frac{8}{11}x - \frac{42}{11}$

9. (a) $y = 10.1x - 79.9$

(b) 122.1 million

2.3 Linear Models

In business and science, it is often necessary to make judgments on the basis of data from the past. For instance, a stock analyst might use a company's profits in previous years to estimate the next year's profits. Or a life insurance company might look at life expectancies of people born in various years to predict how much money it should expect to pay out in the next year.

In such situations, the available data is used to construct a mathematical model, such as an equation or a graph, which is used to approximate the likely outcome in cases where complete data is not available. In this section, we consider applications in which the data can be modeled by a linear equation.

The simplest way to construct a linear model is to use the line determined by two of the data points, as illustrated in the following example.



EXAMPLE 1 **Full-Time Faculty Members** The number of full-time faculty members at four-year colleges and universities (in thousands) in selected years is shown in the following table. (Data from: U.S. National Center for Education Statistics.)

Year	2005	2007	2009	2011	2013
Number of Faculty Members	917	990	1038	1115	1152

Therefore, the estimated higher education enrollment for Michigan would be 567,020 students.

- (c) According to this model, what is the state population that would predict a higher education enrollment of 800 thousand students?

Solution Let $y = 800$ and solve the regression equation for x :

$$800 = .045x + 121.7$$

$$678.3 = .045x$$

$$x \approx 15073.333.$$

Thus, the model predicts that a state with a population of 15,073,333 has an enrollment of 800,000 in higher education.

NOTE The value r^2 seen on the TI calculator output is known as the coefficient of determination. It is simply the value of the correlation coefficient squared and $0 \leq r^2 \leq 1$. One interpretation of its value is that it describes the proportion of variation in the y -variable that is explained by variation in the x -variable.

2.3 Exercises

1. **Temperature** The following table shows equivalent Fahrenheit and Celsius temperatures:

Degrees Fahrenheit	32	68	104	140	176	212
Degrees Celsius	0	20	40	60	80	100

- (a) Choose any two data points and use them to construct a linear equation that models the data, with x being Fahrenheit and y being Celsius.
 (b) Use the model in part (a) to find the Celsius temperature corresponding to

50° Fahrenheit and 75° Fahrenheit.

Temperature Use the linear equation derived in Exercise 1 to work the following problems.

2. Convert each temperature.

- (a) 58°F to Celsius (b) 50°C to Fahrenheit
 (c) -10°C to Fahrenheit (d) -20°F to Celsius

3. According to the *World Almanac and Book of Facts*, 2008, Venus is the hottest planet, with a surface temperature of 867° Fahrenheit. What is this temperature in degrees Celsius?

4. Find the temperature at which Celsius and Fahrenheit temperatures are numerically equal.

In each of the next set of problems, assume that the data can be modeled by a straight line and that the trend continues indefinitely. Use two data points to find such a line and then estimate the requested quantities. (See Example 1.)

5. **CPI** The Consumer Price Index (CPI), which measures the cost of a typical package of consumer goods, was 224.9 in the year 2011 and 240.0 in 2016. Let $x = 11$ correspond to the year 2011 and estimate the CPI in 2012 and 2015. (Data from: U.S. Bureau of Labor Statistics.)

6. **Tax Returns** The approximate number (in millions) of individual electronically filed tax returns filed with the Internal Revenue Service (IRS) in the year 2005 was 68.5, and it was 123.7 in 2016. Let $x = 5$ correspond to the year 2005, and estimate the number of electronically filed returns in 2010 and 2014. *Hint:* Round slope to two decimal places. (Data from: U.S. Internal Revenue Service.)

7. **Educational Services** The number (in millions) of employees working in educational services was 12.3 in 2005 and 13.3 in 2014. Let $x = 5$ correspond to the year 2005, and estimate the number of employees in 2012. *Hint:* Round slope to two decimal places. (Data from: U.S. Census Bureau.)

8. **Leisure and Hospitality** The number (in millions) of employees working in the leisure and hospitality industries was 12.1 in 2005 and 13.5 in 2014. Let $x = 5$ correspond to the year 2005, and estimate the number of employees in 2013. *Hint:* Round slope to two decimal places. (Data from: U.S. Census Bureau.)

9. **Baseball** Suppose a baseball is thrown at 85 miles per hour. The ball will travel 320 feet when hit by a bat swung at 50 miles per hour and will travel 440 feet when hit by a bat swung at 80 miles per hour. Let y be the number of feet traveled by the ball when hit by a bat swung at x miles per hour. (*Note:* The preceding data are valid for $50 \leq x \leq 90$, where the bat is 35 inches long, weighs 32 ounces, and strikes a waist-high pitch so that the place of the swing lies at 10° from the diagonal). [Data from: Robert K. Adair, *The Physics of Baseball* (HarperCollins, 1990)]. How much farther will a ball travel for each mile-per-hour increase in the speed of the bat?

10. **Skiing** Ski resorts require large amounts of water in order to make snow. Snowmass Ski Area in Colorado plans to pump at least 1120 gallons of water per minute for at least 12 hours a day from Snowmass Creek between mid-October and late December. Environmentalists are concerned about the effects on the ecosystem. Find the minimum amount of water pumped in 30 days.

[Hint: Let y be the total number of gallons pumped x days after pumping begins. Note that $(0, 0)$ is on the graph of the equation.] (Data from: York Snow, Inc.)

In each of the next two problems, two linear models are given for the data. For each model,

- find the residuals and their sum;
- find the sum of the squares of the residuals;
- decide which model is the better fit. (See Example 2.)

11. **Cisco Sales** The following table shows the estimated sales (in billions of dollars) in China for Cisco Systems, Inc. for several years.

Year	2006	2008	2010	2012	2014
Sales	.7	1.22	1.53	2.02	1.81

Let $x = 6$ correspond to the year 2006. Two equations that model these data are $y = .07x + .83$ and $y = .2x - .38$ (Data from: *The Wall Street Journal*.)

12. **International Trade** The following table shows South Korea's petrochemical exports to China (in billions of dollars) for several years.

Year	2010	2011	2012	2013	2014
Petrochemical exports	16	21	21	23	21.4

Let $x = 10$ correspond to the year 2010. Two equations that model these data are $y = 1.35x + 2.5$ and $y = 2x - 3$ (Data from: *The Wall Street Journal*.)

In each of the following problems, determine whether a straight line is a good model for the data. Do this visually by plotting the data points and by finding the correlation coefficient for the least-squares regression line. (See Examples 5 and 6.)

13. **Credit Unions** The following table shows the number of operating federal credit unions in the United States for several years.

Year	2011	2012	2013	2014	2015
Number of federal credit unions	4444	4268	4100	3924	3762

Let $x = 11$ correspond to the year 2011. (Data from: Credit Union National Association.)

14. **Gas Prices** The following tables shows the average price (in dollars) for a gallon of regular unleaded gasoline on the first weekday of selected months in the year 2015. Let $x = 1$ correspond to the month of January, $x = 3$ correspond to the month of March, and so on. (Data from: U.S. Energy Information Administration.)

Month	January	March	May	July	September	November
Price per gallon	\$2.14	\$2.36	\$2.52	\$2.70	\$2.34	\$2.17

In Exercises 15–18 find the required linear model using least-squares regression. (See Examples 3–6.)

15. **Credit Unions** Use the data on the number of federal credit unions in Exercise 13.

- Find a linear model for these data with $x = 11$ corresponding to the year 2011.
- Assuming the trend continues, estimate the number of federal credit unions in the year 2018.

16. **Student Loans** The accompanying table shows the approximate amount (in billions of dollars) of student loans in repayment in various quarters of 2015 and 2016. Let $x = 1$ correspond to the first quarter of 2015. (Data from: *The Wall Street Journal*.)

Quarter	1	2	3	4	5	6	7
Loans in repayment	510	525	570	580	610	630	650

- Find a linear model for these data.
- Assuming the trend continues, estimate the amount of loans in repayment in quarter 8.

17. **Google Ad Revenue** The accompanying table shows the approximate amount (in billions of dollars) of Google advertising revenue in various quarters of 2015 and 2016. Let $x = 1$ correspond to the first quarter of 2015. (Data from: *The Wall Street Journal*.)

Quarter	1	2	3	4	5	6	7	8
Ad revenue	15.1	16.2	16.9	18.0	17.5	18.0	19.2	22.4

- Find a linear model for these data.
- Assuming the trend continues, estimate the amount of ad revenue in the third quarter of 2017.

18. **Blood Pressure** Researchers wish to determine if a relationship exists between age and systolic blood pressure (SBP). The accompanying table gives the age and systolic blood pressure for 10 adults.

Age (x)	35	37	42	46	53	57	61	65	69	74
SBP (y)	102	127	120	131	126	137	140	130	148	147

- Find a linear model for these data using age (x) to predict systolic blood pressure (y).
- Use the results from part (a) to predict the systolic blood pressure for a person who is 42-years-old, 53-years-old, and 69-years-old. How well do the actual data agree with the predicted values?
- Use the results from part (a) to predict the systolic blood pressure for a person who is 45-years-old and 70-years-old.

Work these problems.

19. **Street and Highway Construction** The total amount of construction spending on highways and streets (in billions of

(dollars) is given in the accompanying table for the first six months of 2016.

Month	January	February	March	April	May	June
Spending	97	94	94	90	89	88

- (a) Find the least squares regression line that models these data with $x = 1$ corresponding to January 2016.
- (b) Assuming the trend continues, estimate the amount of spending in December 2016.
20. **Cable Subscribers** The number of basic cable television subscribers (in millions) is shown in the following table for various years. (Data from: ProQuest Statistical Abstract of the United States 2016.)

Year	2006	2008	2010	2012	2014
Subscribers	65.8	64.8	61.5	57.8	54.4

- (a) Find the least-squares regression line that models these data with $x = 6$ corresponding to the year 2006.
- (b) Find the number of subscribers for the year 2013.
- (c) If the trend continues indefinitely, determine the first full year when there will be at least 50 million subscribers.
- (d) Find the correlation coefficient.
21. **Software Publishers** The revenue per quarter for the software publishing industry (in billions of dollars) for the 8 quarters of 2015 and 2016 is displayed in the accompanying table. Let $x = 1$ correspond to the first quarter of 2015. (Data from: U.S. Census Bureau.)

Quarter	1	2	3	4	5	6	7	8
Revenue	47.4	48.9	49.8	49.3	49.8	49.8	49.6	50.4

- (a) Find the least-squares regression line that models these data.
- (b) If the trend continues indefinitely, find the revenue for the second quarter of 2017.
- (c) If the trend continues indefinitely, what is the first full quarter when revenue exceeds \$52 billion?
- (d) Find the correlation coefficient.
22. **Life Expectancy** The following table shows men's and women's life expectancy at birth (in years) for selected birth years in the United States. (Data from: U.S. Center for National Health Statistics.)

Birth Year	Life Expectancy	
	Men	Women
1970	67.1	74.7
1975	68.8	76.6
1980	70.0	77.4
1985	71.1	78.2
1990	71.8	78.8
1995	72.5	78.9
2000	74.1	79.3
2005	75.0	80.1
2010	76.2	81.0
2015	77.1	81.7

- (a) Find the least-squares regression line for the men's data, with $x = 70$ corresponding to 1970.
- (b) Find the least-squares regression line for the women's data, with $x = 70$ corresponding to 1970.
- (c) Suppose life expectancy continues to increase as predicted by the equations in parts (a) and (b). Will men's life expectancy ever be higher than women's? If so, in what birth year will this occur?

Checkpoint Answers

- $y = 30.25x + 765.75$
- (a) 30, 29, 3, 6, -31
(b) 2747
(c) Yes, because the sum of the squares of the residuals is lower.
- (a) -6.4, 7.1, -4.4, 13.1, -9.4
(b) 370.7
(c) It fits the data best because the sum of the squares of its residuals is smallest.
- $y = 6.75x + 337.08$
- (a) $y = .87x + 18.9$
(b) $r = -.988$
(c) It fits reasonably well because $|r|$ is very close to 1 and the pattern is linear.

2.4 Linear Inequalities

An **inequality** is a statement that one mathematical expression is greater than (or less than) another. Inequalities are very important in applications. For example, a company wants revenue to be *greater than* costs and must use *no more than* the total amount of capital or labor available.

Inequalities may be solved by algebraic or geometric methods. In this section, we shall concentrate on algebraic methods for solving **linear inequalities**, such as

$$4 - 3x \leq 7 + 2x \quad \text{and} \quad -2 < 5 + 3m < 20.$$

Solve each part separately:

$$-7m < -7 \quad \text{or} \quad -7m > 3$$

$$m > 1 \quad \text{or} \quad m < -\frac{3}{7}$$

Checkpoint 8

Solve each inequality. Graph each solution.

(a) $|y - 2| > 5$

(b) $|3k - 1| \geq 2$

(c) $|2 + 5r| - 4 \geq 1$

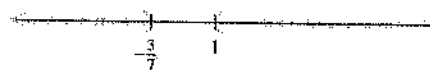
The solution, all numbers in $(-\infty, -\frac{3}{7})$ or $(1, \infty)$, is graphed in Figure 2.52.

Figure 2.52

Checkpoint 9

Solve each inequality.

(a) $|5m - 3| > -10$

(b) $|6 + 5a| < -9$

(c) $|8 + 2r| > 0$

Solve $|3 - 7x| \geq -8$.

Solution The absolute value of a number is always nonnegative. Therefore, $|3 - 7x| \geq -8$ is always true, so the solution is the set of all real numbers. Note that the inequality $|3 - 7x| \geq -8$ has no solution, because the absolute value of a quantity can never be less than a negative number.

2.4 Exercises

1. Explain how to determine whether a parenthesis or a bracket is used when graphing the solution of a linear inequality.
2. The three-part inequality $p < x < q$ means " p is less than x and x is less than q ." Which one of the given inequalities is not satisfied by any real number x ? Explain why.
- (a) $-3 < x < 5$ (b) $0 < x < 4$
(c) $-7 < x < -10$ (d) $-3 < x < -2$

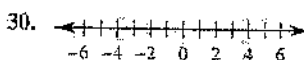
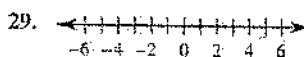
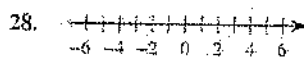
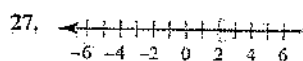
Solve each inequality and graph each solution. (See Examples 1–3.)

3. $-8k \leq 32$ 4. $-4a \leq 36$
5. $-2b > 0$ 6. $6 - 6z < 0$
7. $3x + 4 \leq 14$ 8. $2y - 7 < 9$
9. $-5 - p \geq 3$ 10. $5 - 3r \leq -4$
11. $7m - 5 < 2m + 10$ 12. $6x - 2 > 4x - 10$
13. $m - (4 + 2m) + 3 < 2m + 2$
14. $2p - (3 - p) \leq -7p - 2$
15. $-2(3y - 8) \geq 5(4y - 2)$
16. $5r - (r + 2) \geq 3(r - 1) + 6$
17. $3p - 1 < 6p + 2(p - 1)$
18. $x + 5(x + 1) > 4(2 - x) + x$
19. $-7 < y - 2 < 5$ 20. $-3 < m + 6 < 2$
21. $8 \leq 3r + 1 \leq 16$ 22. $-6 < 2p - 3 \leq 5$
23. $-4 \leq \frac{2k - 1}{3} \leq 2$ 24. $-1 \leq \frac{5y + 2}{3} \leq 4$

25. $\frac{3}{5}(2p + 3) \geq \frac{1}{10}(5p + 1)$

26. $\frac{8}{3}(z - 4) \leq \frac{2}{9}(3z + 2)$

In the following exercises, write a linear inequality that describes the given graph.



Break-Even Point In Exercises 31–36, find all values of x for which the given products will at least break even. (See Examples 5 and 6.)

31. $C = 50x + 6000$; $R = 65x$
32. $C = 100x + 6000$; $R = 500x$
33. $C = 85x + 1000$; $R = 105x$
34. $C = 70x + 500$; $R = 60x$
35. $C = 1000x + 5000$; $R = 900x$
36. $C = 25,000x + 21,700,000$; $R = 102,500x$

Solve each inequality. Graph each solution. (See Examples 7–10.)

37. $|p| > 7$ 38. $|m| < 2$
39. $|r| \leq 5$ 40. $|a| < -2$

41. $x - 5 \leq 7$
 42. $|2x + 5| < 1$
 43. $|x - \frac{1}{2}| \geq 2$
 44. $|3z + 1| \geq 4$
 45. $x^2 + 5 \leq 7$
 46. $|5x + \frac{1}{2}| - 2 < 5$

Work these problems.

Temperature The given inequality describes the monthly average high-daily temperature T in degrees Fahrenheit in the given location. (Data from: Weatherbase.com.) What range of temperatures corresponds to the inequality?

47. $T - 83 \leq 7$; Miami, Florida
 48. $T - 62 \leq 27$; Boise, Idaho
 49. $T - 61 \leq 21$; Flagstaff, Arizona
 50. $T - 43 \leq 22$; Anchorage, Alaska

51. **Engagement at Work** According to the polling organization Gallup.com, 35.5% of American workers are meaningfully engaged at work. Taking into account the margin of error, we can write this result as $|P - 35.5| \leq 3$, where P is the percentage of American workers engaged at work. Find the range of estimates for P .

52. **Social Science** When administering a standard intelligence quotient (IQ) test, we expect about one-third of the scores to be more than 12 units above 100 or more than 12 units below 100. Describe this situation by writing an absolute value inequality.

53. **Social Science** A Gallup poll found that among Americans aged 18–34 years who consume alcohol, from 35% to 43% prefer beer. Let B represent the percentage of American alcohol consumers who prefer beer. Write the preceding information as an inequality. (Data from: Gallup.com.)

54. **Social Science** A Gallup poll in January 2017 found that from 13.1% to 15.1% of Americans considered themselves underemployed (working part-time or not working at all, but wanting to work full-time). Let U represent the percentage of underemployed workers. Write the preceding information as an inequality. (Data from: Gallup.com.)

55. **Income Tax** The following table shows the 2017 federal income tax for a single person. (Data from: Internal Revenue Service.)

If Taxable Income Is Over	But Not Over	Tax Rate Is
\$0	\$9325	10%
\$9325	\$37,950	15%
\$37,950	\$91,900	25%
\$91,900	\$191,650	28%
\$191,650	\$416,700	33%
\$416,700	\$418,400	35%
\$418,400	No Limit	39.6%

Let x denote the taxable income. Write each of the seven income ranges in the table as an inequality.

56. **Income Tax** The following table shows the 2016 New York state income tax for a single person. (Data from: www.tax-brackets.org.)

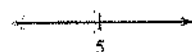
If Taxable Income Is Over	But Not Over	Tax Rate Is
\$0	\$8450	4.00%
\$8450	\$11,650	4.50%
\$11,650	\$13,850	5.25%
\$13,850	\$21,300	5.90%
\$21,300	\$80,150	6.45%
\$80,150	\$214,000	6.65%
\$214,000	\$1,070,350	6.85%
\$1,070,350	No Limit	8.82%

Let x denote the taxable income. Write each of the eight income ranges in the table as an inequality.

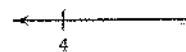
Checkpoint Answers

1. (a) $-24 < -4$; $42 > 7$ (b) $18 \geq -8$; $-45 \leq 20$
 (c) $1 < 3$; $-9 < -7$

2. (a) $z < 5$



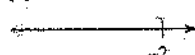
- (b) $k \geq 4$



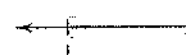
- (c) $y \leq -4$



3. (a) $t \leq -2$



- (b) $r > 1$



4. (a) $4 < k < 8$



- (b) $-5 \leq z \leq 4$



5. $15^\circ\text{C to } 35^\circ\text{C}$

6. (a) $[-1, 1]$



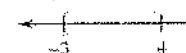
- (b) All numbers in $(-\infty, -3]$ or $[3, \infty)$



7. (a) $(-7, 1)$



- (b) $[-3, 4]$



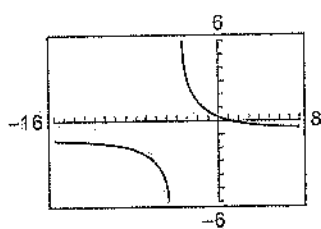


Figure 2.62

values of x that make the numerator and denominator 0 (namely, $x = 1$ and $x = -4$). Then graph $\frac{1-x}{x+4}$. Figure 2.62 shows that the graph is above the x -axis when it is between the vertical asymptote at $x = -4$ and the x -intercept at $x = 1$. So the solution of the inequality is the interval $(-4, 1)$. (When the values that make the numerator and denominator 0 cannot be found algebraically, as they were here, you can use the root finder to approximate them.)

2.5 Exercises

Solve each of these quadratic inequalities. Graph the solutions on the number line. (See Examples 2 and 3.)

1. $(x+4)(2x-3) \leq 0$
2. $(5y-1)(y+3) > 0$
3. $r^2 + 4r > -3$
4. $z^2 + 6z > -8$
5. $4m^2 + 7m - 2 \leq 0$
6. $6p^2 - 11p + 3 \leq 0$
7. $4x^2 + 3x - 1 > 0$
8. $3x^2 - 5x > 2$
9. $x^2 \leq 36$
10. $y^2 \geq 9$
11. $p^2 - 16p > 0$
12. $r^2 - 9r < 0$

Solve these inequalities. (See Example 4.)

13. $x^3 - 9x \geq 0$
14. $p^3 - 25p \leq 0$
15. $(x+7)(x+2)(x-2) \geq 0$
16. $(2x+4)(x^2-9) \leq 0$
17. $(x+5)(x^2-2x-3) < 0$
18. $x^3 - 2x^2 - 3x \leq 0$
19. $6k^3 - 5k^2 < 4k$
20. $2m^3 + 7m^2 > 4m$
21. A student solved the inequality $p^2 < 16$ by taking the square root of both sides to get $p < 4$. She wrote the solution as $(-\infty, 4)$. Is her solution correct?

Use a graphing calculator to solve these inequalities. (See Example 5.)

22. $6x + 7 < 2x^2$
23. $5x^2 - 1.2x < .2$
24. $3.1x^2 - 7.4x + 3.2 > 0$
25. $x^3 - 2x^2 - 5x + 7 \geq 2x + 1$
26. $x^4 - 6x^3 + 2x^2 < 5x - 2$
27. $2x^4 + 3x^3 < 2x^2 + 4x - 2$
28. $x^5 + 5x^4 > 4x^3 - 3x^2 - 2$

Solve these rational inequalities. (See Examples 7 and 8.)

29. $\frac{r-4}{r-1} \leq 0$
30. $\frac{z+6}{z+4} > 1$
31. $\frac{a-2}{a-5} < -1$
32. $\frac{1}{3k-5} < \frac{1}{3}$
33. $\frac{1}{p-2} < \frac{1}{3}$
34. $\frac{7}{k+2} \geq \frac{1}{k+2}$

$$35. \frac{5}{p+1} > \frac{12}{p+1}$$

$$36. \frac{x^2-4}{x} > 0$$

$$37. \frac{x^2-x-6}{x} < 0$$

38. Determine whether $x+4$ is positive or negative when

(a) $x > -4$; (b) $x < -4$.

(c) If you multiply both sides of the inequality $\frac{1-x}{x+4} \geq 0$ by $x+4$, should you change the direction of the inequality sign? If so, when?

(d) Explain how you can use parts (a)–(c) to solve $\frac{1-x}{x+4} \geq 0$ correctly.

Use a graphing calculator to solve these inequalities. You may have to approximate the roots of the numerators or denominators.

$$39. \frac{2x^2+x-1}{x^2-4x+4} \leq 0$$

$$40. \frac{x^3-3x^2+5x-29}{x^2-7} > 3$$

41. Profit An analyst has found that her company's profits, in hundreds of thousands of dollars, are given by $P = 2x^2 - 12x - 32$, where x is the amount, in hundreds of dollars, spent on advertising. For what values of x does the company make a profit?

42. Profit The commodities market is highly unstable; money can be made or lost quickly on investments in soybeans, wheat, and so on. Suppose that an investor kept track of his total profit P at time t , in months, after he began investing, and he found that $P = 4t^2 - 30t + 14$. Find the time intervals during which he has been ahead.

43. Profit The manager of a 200-unit apartment complex has found that the profit is given by

$$P = x^2 + 300x - 18,000,$$

where x is the number of apartments rented. For what values of x does the complex produce a profit?

44. Profit A door-to-door knife salesman finds that his weekly profit can be modeled by the equation

$$P = x^2 + 5x - 530$$

where x is the number of pitches he makes in a week. For what values of x does the salesman need to make in order to earn a profit?